# Solid-Liquid Phase Equilibria of Binary and Ternary Mixtures of Benzene and Polynuclear Aromatic Compounds 

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#### Abstract

Solid-liquid equilibrium phase diagrams for three binary miztures of benzene with polynuclear aromatic compounds (fluorene, dibenzofuran, and dibenzothiophene) have been measured by a dynamic method in the temperature range from 0 to $80^{\circ} \mathrm{C}$. These binary data plus previously published data derived from calorimetric studies were used to determine experimental activity coefficients and interaction parameters using the Wilson and UNIQUAC models. The resulting set of interaction parameters gave a good representation of the experimental binary data and ternary data at $50^{\circ} \mathrm{C}$, but were not generally satisfactory for predicting solid-liquid equilibrium in ternary systems at lower temperatures.


## Introduction

Separation of organic close-boiling or close-melting components from a mirture is a challenging task in some chemical engineering processes, such as crystallization and extraction. With the present trend in the petroleum industry toward heavier feedstocks and coal-derived liquids, there is a strong need for expansion of the small database of thermodynamic data on heavier compounds that presently exists. The present work is a continuation of systematic studies on solid-liquid equilibrium (SLE) in binary and ternary systems of polynuclear aromatic compounds (1-7). There are very few data available for ternary aromatic systems consisting of solvent-solid-solid at different temperatures. Some results of this kind are reported in refs 8-10, for example. In our previous study (6), we examined the phase equilibria of two ternary systems containing cis-decalin, naphthalene, and biphenyl and tetralin, biphenyl, and dibenzofuran at $25^{\circ} \mathrm{C}$. The first component in each of these systems was a liquid, and the other two components were solids. These ternary systems included three pairs of binary eutectics. The ternary phase diagrams were also predicted using binary parameters obtained from regression of binary data using the UNIQUAC model.

The purpose of the present study was to select systems with a solid solution on one side of the phase diagram. In this paper we present solid-liquid phase equilibria of three ternary systems of this kind: benzene, fluorene, dibenzofuran; benzene, fluorene, dibenzothiophene; and benzene, dibenzothiophene, dibenzofuran, at temperatures from 0 to $80^{\circ} \mathrm{C}$. As indicated by Sediawan et al. (1), the fluorene-dibenzofuran system forms a solid solution at any composition, while the fluorenedibenzothiophene and dibenzothiophene-dibenzofuran systems have solid-phase immiscibility gaps.

New SLE data are also reported for three binary systems: mixtures of benzene with fluorene, dibenzofuran, and dibenzothiophene.

An additional goal of this study is to predict solid-liquid equilibria (SLE) for ternary systems containing such polynuclear aromatic compounds by using binary Wilson or UNIQUAC parameters obtained from the regression of binary SLE data.
The first section is a discussion of the procedure for the solid-liquid equilibrium experiments. This is followed by a

[^0]brief discussion of the method of determining the Wilson and UNIQUAC constants and correlating results for binary systems. The results of the ternary system analysis are presented in the third section. The last section discusses some of the important conclusions observed in this study. We hope these data will be useful in industry for design purposes as well as in the development of solid-liquid equilibrium theories.

## Experimental Section

The chemicals used were from Aldrich Chemicals, $99 \%$ pure grade. The fluorene, dibenzofuran, and dibenzothiophene were recrystallized from toluene. They were analyzed by gas chromatography using a phenyl methyl silicone capillary column and flame ionization detector. Purities (area \%) were $99.7 \%$ for fluorene, $99.8 \%$ for dibenzofuran, and $99.4 \%$ for dibenzothiophene. Benzene was purified by fractional distillation and stored over 4A molecular sieves.

Solubilities were determined by a dynamic (synthetic) method described in full by Domanska (7). Mixtures of solute and solvent in binary systems and of two solutes and solvent (benzene) in ternary systems, prepared by weighing, were heated very slowly (heating rate did not exceed $2^{\circ} \mathrm{C} \mathrm{h}^{-1}$ near the equilibrium temperature) with stirring. The temperature at which the last crystals disappeared (decline of solution cloudiness) was taken as the temperature of the solutioncrystal equilibrium. Measurements were performed over a wide range of solute concentration and over the temperature range from $0^{\circ} \mathrm{C}$ to the boiling temperature of benzene ( 80.3 ${ }^{\circ} \mathrm{C}$ ). The accuracy of temperature measurements was $\pm 0.1$ ${ }^{\circ} \mathrm{C}$. The mixture composition was known within $\pm 0.0003$ mole fraction. The saturation temperature for a given mixture was reproducible within $\pm 0.1^{\circ} \mathrm{C}$.
The characteristics of the compounds are collected in Table I. All the direct experimental data are shown in Tables II-V.

Table I. Thermodynamic Properties of Solutes

| solute | ${ }^{t_{m} /}{ }^{\text {c }}$ | $\underset{\left(\mathrm{J} \mathrm{~mol}^{-1}\right)}{\Delta H_{0}}$ | $\left(\mathrm{J} \mathrm{~mol}^{-1}\right)$ | $\begin{gathered} V^{2 s} / \\ \left(\mathrm{cm}^{3} \mathrm{~mol}^{-1}\right) \end{gathered}$ | ${ }^{t_{0}{ }^{\circ} \mathrm{C}}$ | $\underset{(\mathrm{J} \text { mol }}{\left.\Delta H_{\mathrm{l}}{ }^{-1}\right)}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| benzene | 5.50 | $9866^{\circ}$ |  | 89.4 |  |  |
| fluorene | 114.75 | $19200^{\circ}$ | $1.98{ }^{\text {b }}$ | 163.7 |  |  |
| dibenzofuran | 82.15 | $1860{ }^{\circ}$ | $9.60{ }^{\circ}$ | 170.0 |  |  |
| dibenzothiophene | 98.80 | $21000^{\circ}$ | $31.40^{\circ}$ | 170.1 | 88.8 | 1500 |

Table II. Experimental Mole Fraction Solubilities and Activity Coefficients for the Three Solutes in Benzene

| $\begin{aligned} & \text { fluorene (1)- } \\ & \text { benzene (2) } \end{aligned}$ |  |  | dibenzofuran (1)benzene (2) |  |  | dibenzothiophene (1)benzene (2) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $x_{1}$ | $t /{ }^{\circ} \mathrm{C}$ | $\gamma_{1}$ | $x_{1}$ | $t /{ }^{\circ} \mathrm{C}$ | $\gamma_{1}$ | $x_{1}$ | $t /{ }^{\circ} \mathrm{C}$ | $\gamma_{1}$ |
| 0.0000 | 5.5 |  | 0.0000 | 5.5 |  | 0.0000 | 5.5 |  |
| 0.0202 | 4.3 | $1.002^{\text {a }}$ | 0.0635 | 2.9 | $1.027^{a}$ | 0.0230 | 4.8 | $1.013^{\circ}$ |
| 0.0407 | 3.8 | $1.008^{\circ}$ | 0.0856 | 0.8 | $1.017^{\circ}$ | 0.0430 | 4.0 | $1.021^{\text {a }}$ |
| 0.0535 | 2.6 | $1.014^{\text {a }}$ | 0.0934 | 0.2 | $1.015^{\text {a }}$ | 0.0532 | 3.4 | $1.022^{\text {a }}$ |
| 0.0631 | 2.1 | 1.321 | 0.1122 | -0.8 | 1.369 | 0.0598 | 2.8 | 1.898 |
| 0.0714 | 4.9 | 1.314 | 0.1239 | 1.6 | 1.328 | 0.0663 | 5.0 | 1.821 |
| 0.0826 | 9.9 | 1.306 | 0.1363 | 4.7 | 1.317 | 0.0784 | 9.9 | 1.764 |
| 0.0986 | 15.7 | 1.294 | 0.1463 | 7.0 | 1.311 | 0.0867 | 12.5 | 1.712 |
| 0.1040 | 17.5 | 1.290 | 0.1599 | 9.6 | 1.288 | 0.1010 | 17.3 | 1.670 |
| 0.1181 | 21.8 | 1.279 | 0.1978 | 17.1 | 1.264 | 0.1152 | 21.5 | 1.633 |
| 0.1301 | 25.2 | 1.271 | 0.2346 | 23.1 | 1.240 | 0.1330 | 26.0 | 1.586 |
| 0.1548 | 31.5 | 1.242 | 0.2480 | 25.1 | 1.232 | 0.1429 | 28.1 | 1.556 |
| 0.1723 | 35.5 | 1.221 | 0.2813 | 29.3 | 1.202 | 0.1573 | 31.1 | 1.522 |
| 0.2062 | 42.0 | 1.189 | 0.2995 | 32.1 | 1.206 | 0.1699 | 33.5 | 1.494 |
| 0.2609 | 51.3 | 1.160 | 0.3559 | 38.8 | 1.183 | 0.2066 | 39.7 | 1.425 |
| 0.3144 | 58.4 | 1.140 | 0.4486 | 46.8 | 1.119 | 0.2516 | 46.0 | 1.354 |
| 0.3565 | 64.3 | 1.113 | 0.5791 | 55.9 | 1.051 | 0.3038 | 52.4 | 1.295 |
| 0.3792 | 66.5 | 1.110 | 0.7045 | 64.5 | 1.023 | 0.3601 | 57.5 | 1.222 |
| 0.4253 | 71.3 | 1.099 | 0.7570 | 67.7 | 1.014 | 0.4131 | 62.2 | 1.178 |
| 0.4538 | 74.5 | 1.094 | 0.8346 | 73.2 | 1.020 | 0.4733 | 67.6 | 1.152 |
| 0.4688 | 76.0 | 1.000 | 1.0000 | 82.1 | 1.000 | 0.5351 | 73.2 | 1.142 |
| 1.0000 | 114.8 |  |  |  |  | 0.5858 | 77.1 | 1.129 |
|  |  |  |  |  |  | 0.6423 | 80.0 | 1.090 |
|  |  |  |  |  |  | 1.0000 | 98.8 | 1.000 |

${ }^{a}$ Activity coefficients for benzene as a solute.

## Results and Discussion

The solubility of solid 1 in a liquid may be expressed by (12)

$$
\begin{array}{r}
\ln x_{1}=\frac{\Delta H_{\mathrm{m} 1}}{R}\left(\frac{1}{T_{\mathrm{t} 1}}-\frac{1}{T}\right)-\frac{\Delta C_{p \mathrm{~m} 1}}{R}\left(\ln \frac{T_{\mathrm{t} 1}}{T}-\frac{T_{\mathrm{t} 1}}{T}+1\right)+ \\
\frac{\Delta H_{\mathrm{tr} 1}}{R}\left(\frac{1}{T_{\mathrm{tr} 1}}-\frac{1}{T}\right)-\ln \gamma_{1} \tag{1}
\end{array}
$$

where $x_{1}, \gamma_{1}, \Delta H_{\mathrm{m} 1}, \Delta C_{p \mathrm{~m} 1}, T_{\mathrm{t} 1}$, and $T$ stand for the mole fraction, activity coefficient, enthalpy of fusion, solute heat capacity difference between the solid and the liquid at the melting point, triple point temperature of the solute, and equilibrium temperature, respectively, and $\Delta H_{\mathrm{tr}}$ and $T_{\mathrm{tr} 1}$ stand for enthalpy of transition and transition temperature of the solute.

For the organic compounds of this study it is an excellent approximation to substitute the normal melting point temperature for the triple point temperature. The term involving the transition enthalpy and temperature is only needed when the solution temperature is below a solid-phase transition temperature.

In this work the activity coefficient was calculated by eq 1 from the experimentally determined solution composition and temperature. Activity coefficients obtained in this way from binary data were used to determine Wilson (13) and UNIQUAC (14) interaction parameters. The Wilson and UNIQUAC equations were then used to predict the equilibrium data for the ternary mixtures.

The parameters were fitted to the binary data by an optimization technique. The objective function was as follows:

$$
\begin{equation*}
F\left(A_{1}, A_{2}\right)=\sum_{i=1}^{n} w_{i}^{-2}\left[\ln \left(x_{1 i} \gamma_{1 i}\left(T, x_{1 i}, A_{1}, A_{2}\right)\right)-\ln a_{1 i}\left(T_{i}\right)\right]^{2} \tag{2}
\end{equation*}
$$

where $\ln a_{1 i}$ denotes an experimental value of the logarithm of solute activity ( $x_{1} \gamma_{1}$, from eq 1 ), $w_{i}$ is the weight of an experimental point, $A_{1}$ and $A_{2}$ are the two adjustable parameters of the correlation equations, $i$ denotes the $i$ th experimental point, and $n$ is the number of experimental

Table III. Equilibrium Data for the Ternary System Benzene (1)-Fluorene (2)-Dibenzofuran (3)A

| $x_{1}$ | $t /{ }^{\circ} \mathrm{C}$ | $x_{1}$ | $t /{ }^{\circ} \mathrm{C}$ | $x_{1}$ | $t /{ }^{\circ} \mathrm{C}$ | $x_{1}$ | $t /{ }^{\circ} \mathrm{C}$ | $x_{1}$ | $t /{ }^{\circ} \mathrm{C}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $x_{2}{ }^{\text {o }}=0.0866$ |  | $x_{2}{ }^{0}=0.1469$ |  | $x_{2}{ }^{\circ}=0.2046$ |  | $x_{2}{ }^{0}=0.2437$ |  | $x_{2}{ }^{\circ}=0.2520$ |  |
| 0.8220 | 12.0 | 0.8915 | 11.0 | 0.8178 | 10.4 | 0.8152 | 11.9 | 0.8003 | 15.2 |
| 0.7905 | 16.8 | 0.7857 | 16.9 | 0.7984 | 13.8 | 0.8008 | 15.0 | 0.7452 | 24.5 |
| 0.7555 | 22.0 | 0.7495 | 22.4 | 0.7773 | 18.1 | 0.7792 | 18.9 | 0.7193 | 28.6 |
| 0.7248 | 27.4 | 0.6487 | 35.8 | 0.7516 | 22.2 | 0.7288 | 27.5 | 0.6824 | 33.6 |
| 0.7027 | 29.8 | 0.5723 | 43.1 | 0.6795 | 32.1 | 0.6390 | 40.0 | 0.6423 | 38.8 |
| 0.6799 | 32.7 | 0.5051 | 50.0 | 0.6189 | 38.9 | 0.6008 | 43.4 | 0.5728 | 46.8 |
| 0.6549 | 35.6 | 0.3910 | 60.0 | 0.5775 | 44.1 | 0.5351 | 50.0 | 0.5251 | 51.7 |
| 0.6179 | 39.6 |  |  | 0.5281 | 49.4 | 0.4350 | 60.0 | 0.4310 | 61.2 |
| 0.5010 | 50.0 |  |  | 0.4750 | 54.6 |  |  |  |  |
| 0.4351 | 55.7 |  |  | 0.4080 | 61.2 |  |  |  |  |
| 0.3720 | 60.0 |  |  |  |  |  |  |  |  |
| $x_{2}{ }^{0}=0.2939$ |  | $x_{2}{ }^{0}=0.3397$ |  | $x_{2}{ }^{0}=0.3736$ |  | $x_{2}{ }^{\circ}=0.3979$ |  | $x_{2}{ }^{\circ}=0.4537$ |  |
| 0.8198 | 10.4 | 0.8042 | 15.3 | 0.8158 | 13.7 | 0.8311 | 11.9 | 0.8172 | 16.7 |
| 0.7652 | 20.4 | 0.7849 | 18.4 | 0.7943 | 18.0 | 0.8124 | 15.7 | 0.7825 | 23.3 |
| 0.7168 | 27.7 | 0.7384 | 26.4 | 0.7621 | 23.1 | 0.7940 | 19.1 | 0.7604 | 27.2 |
| 0.6972 | 30.0 | 0.7164 | 29.2 | 0.7306 | 30.0 | 0.7786 | 21.9 | 0.7092 | 35.1 |
| 0.6345 | 38.1 | 0.6676 | 36.3 | 0.6781 | 35.8 | 0.7612 | 24.8 | 0.6663 | 40.6 |
| 0.6050 | 42.0 | 0.6294 | 40.8 | 0.6368 | 40.2 | 0.7357 | 28.8 |  |  |
| 0.5601 | 47.0 | 0.5477 | 50.8 |  |  | 0.6929 | 34.6 |  |  |
| 0.4922 | 53.9 |  |  |  |  | 0.6270 | 42.8 |  |  |
| 0.4220 | 60.6 |  |  |  |  | 0.5710 | 48.8 |  |  |
| $x_{2}{ }^{0}=0.5003$ |  | $x_{2}{ }^{0}=0.5566$ |  | $x_{2}{ }^{\circ}=0.6005$ |  | $x_{2}{ }^{\text {a }}=0.6719$ |  | $x_{2}{ }^{0}=0.7262$ |  |
| 0.8367 | 14.7 | 0.8632 | 11.4 | 0.8573 | 14.0 | 0.8742 | 13.2 | 0.8730 | 15.5 |
| 0.8235 | 17.3 | 0.8257 | 19.9 | 0.8383 | 18.5 | 0.8574 | 18.1 | 0.8519 | 20.9 |
| 0.8059 | 21.4 | 0.8040 | 24.1 | 0.8193 | 22.5 | 0.8410 | 21.1 | 0.8410 | 23.2 |
| 0.7703 | 27.5 | 0.7830 | 27.8 | 0.7945 | 27.4 | 0.8051 | 27.8 | 0.8146 | 29.0 |
| 0.7405 | 32.1 | 0.7601 | 31.4 | 0.7654 | 32.2 | 0.7680 | 34.5 | 0.7850 | 35.0 |
| 0.6105 | 49.7 | 0.7260 | 36.8 | 0.7394 | 36.3 | 0.7498 | 37.4 | 0.7281 | 43.1 |
|  |  | 0.6830 | 42.6 | 0.7041 | 41.5 |  |  | 0.6676 | 51.0 |
|  |  | 0.6118 | 50.7 | $0.6494$ | 48.4 |  |  | $0.6003$ | $61.7$ |
|  |  |  |  | 0.5689 | 57.6 |  |  | 0.4734 | 80.0 |
| $x_{2}{ }^{\circ}=0.7600$ |  | $x_{2}{ }^{\circ}=0.8086$ |  | $x_{2}{ }^{\circ}=0.8681$ |  | $x_{2}{ }^{0}=0.9138$ |  |  |  |
| 0.8784 | 14.5 | 0.8808 | 15.8 | 0.8821 | 16.0 | 0.9113 | 9.5 |  |  |
| 0.8373 | 24.6 | 0.8646 | 20.2 | 0.8585 | 24.9 | 0.8796 | 20.1 |  |  |
| 0.8051 | 31.1 | 0.8501 | 23.8 | 0.8352 | 30.3 | 0.8452 | 29.1 |  |  |
| 0.7654 | 39.2 | 0.8372 | 26.8 | 0.8271 | 32.5 | 0.8121 | 35.8 |  |  |
| 0.7167 | 46.0 | 0.8202 | 30.6 | 0.8002 | 37.5 | 0.7656 | 44.1 |  |  |
| 0.6781 | 51.2 | 0.8093 | 33.0 | 0.7890 | 41.5 | 0.6660 | 56.2 |  |  |
| 0.6314 | 56.6 | 0.7775 | 38.5 | 0.7631 | 43.4 | 0.5871 | 66.2 |  |  |
| 0.5990 | 61.4 | 0.7642 | 41.1 | 0.6903 | 53.4 | 0.4801 | 79.1 |  |  |
| 0.5085 | 70.9 | $0.6757$ | $53.3$ | 0.6170 | 62.5 |  |  |  |  |
|  |  | $0.5842$ | $64.0$ |  |  |  |  |  |  |

${ }^{a} x_{2}{ }^{0}=$ mole fraction of the fluorene in the benzene-free binary solid mixtures.
points. The weights were calculated by means of the error propagation formula:

$$
\begin{align*}
& w_{i}^{2}=\left(\frac{\partial \ln x_{1} \gamma_{1}-\partial \ln a_{i}}{\partial T}\right)_{T=T_{i}}^{2}\left(\Delta T_{i}\right)^{2}+ \\
&\left(\frac{\partial \ln x_{1} \gamma_{1}}{\partial x_{1}}\right)_{x_{1}=x_{1 i}}^{2}\left(\Delta x_{1 i}\right)^{2} \tag{3}
\end{align*}
$$

where $\Delta T$ and $\Delta x_{1}$ are the estimated errors of $T$ and $x_{1}$, respectively.

According to the above formulation, the objective function is consistent with the maximum likelihood principle, provided that the first-order approximation (eq 3) is valid. Neau and Peneloux (15) called this procedure the observed deviation method.
The experimental errors of temperature and solute mole fraction were fixed for all cases and set to $\Delta T=0.1 \mathrm{~K}$ and $\Delta x_{1}=0.001$.
The root mean square deviation of temperature defined by eq 4, where $T_{i}$ calcd and $T_{i}$ are, respectively, the calculated and experimental temperatures of the $i$ th point and $n$ is the number of experimental points, was used as a measure of the

$$
\begin{equation*}
\sigma_{T}=\left[\sum_{i=1}^{n} \frac{\left(T_{i}^{\text {calcd }}-T_{i}\right)^{2}}{(n-2)}\right]^{1 / 2} \tag{4}
\end{equation*}
$$

goodness of fit of the solubility curves. The calculated values

Table IV. Equilibrium Data for the Ternary System Benzene (1)-Fluorene (2)-Dibenzofuran (3)d

| $x_{1}$ | $t /{ }^{\circ} \mathrm{C}$ | $x_{1}$ | $t /{ }^{\circ} \mathrm{C}$ | $x_{1}$ | $t /{ }^{\circ} \mathrm{C}$ | $x_{1}$ | $t /{ }^{\circ} \mathrm{C}$ | $x_{1}$ | $t /{ }^{\circ} \mathrm{C}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $x_{2}{ }^{\circ}=0.0290$ |  | $x_{2}{ }^{\circ}=0.0829$ |  | $x_{2}{ }^{\circ}=0.1221$ |  | $x_{2}{ }^{\circ}=0.1391$ |  | $x_{2}{ }^{0}=0.1743$ |  |
| 0.9100 | 13.5 | $0.9022$ | $14.0$ | $0.8903 \quad 16.1$ |  | 0.9042 | 11.4 | $0.9062 \quad 9.6$ |  |
| 0.8478 | 29.6 | 0.8904 | 17.6 | 0.8553 | 24.4 | 0.8832 | 17.2 | 0.8580 | 22.1 |
| 0.7740 | 42.7 | 0.8660 | 23.8 | 0.8405 | 27.5 | 0.8650 | 21.4 | 0.8313 | 27.4 |
| 0.7341 | 48.0 | 0.8259 | 32.3 | 0.7782 | 38.0 | 0.8382 | 27.0 | 0.7977 | 33.4 |
| 0.6571 | 57.3 | 0.7843 | 39.2 | 0.7375 | 43.3 | 0.8204 | 30.2 | 0.7631 | 42.5 |
| 0.5631 | 67.1 | 0.7479 | 44.6 | 0.6136 | 56.7 | 0.7913 | 35.0 | 0.6520 | 52.0 |
|  |  | 0.7057 | 50.3 | 0.5365 | 63.3 | 0.7592 | 39.7 | 0.5117 | 66.8 |
|  |  | 0.6364 | 58.0 | 0.4642 | 69.1 | 0.7180 | 45.2 | 0.4118 | 77.6 |
|  |  | 0.5803 | 62.4 |  |  | 0.6861 | 48.9 |  |  |
|  |  | 0.5002 | 70.0 |  |  | 0.6029 | 56.9 |  |  |
|  |  |  |  |  |  | 0.5521 | 60.8 |  |  |
| $x_{2}{ }^{0}=0.2008$ |  | $x_{2}{ }^{\circ}=0.2083$ |  | $x_{2}{ }^{\circ}=0.2838$ |  | $x_{2}{ }^{\circ}=0.3083$ |  | $x_{2}{ }^{0}=0.3403$ |  |
| 0.8906 | 13.5 | 0.8815 | 15.9 | 0.8725 | 15.0 | 0.8831 | 13.2 | 0.8861 | 12.3 |
| 0.8341 | 26.1 | 0.8587 | 21.1 | 0.8623 | 17.5 | 0.8359 | 24.2 | 0.8456 | 23.3 |
| 0.8201 | 28.8 | 0.8329 | 26.7 | 0.8461 | 21.7 | 0.8023 | 30.7 | 0.8165 | 28.5 |
| 0.7821 | 35.0 | 0.8065 | 31.4 | 0.8285 | 25.4 | 0.7444 | 40.4 | 0.7489 | 40.1 |
| 0.6724 | 49.7 | 0.7733 | 36.6 | 0.8210 | 27.0 | 0.6987 | 46.3 | 0.6906 | 48.3 |
| 0.5824 | 60.2 | 0.7450 | 40.6 | 0.7900 | 32.4 | 0.6355 | 54.1 | 0.6379 | 54.8 |
| 0.4614 | 73.9 | 0.6784 | 50.0 | 0.7740 | 35.3 | 0.5331 | 63.8 | 0.5432 | 64.5 |
|  |  | 0.6319 | 56.2 | 0.5856 | 59.6 | 0.4701 | 70.0 | 0.4835 | 69.3 |
|  |  | 0.4817 | 74.2 | 0.5108 | 67.0 |  |  |  |  |
| $x_{2}{ }^{0}=0.3718$ |  | $x_{2}{ }^{\circ}=0.4210$ |  | $x_{2}{ }^{\circ}=0.4938$ |  | $x_{2}{ }^{0}=0.5474$ |  | $x_{2}{ }^{0}=0.6048$ |  |
| 0.8904 | 12.0 | 0.8800 | 14.5 | 0.8915 | 12.5 | 0.8847 | 14.9 | 0.8937 | 12.8 |
| 0.8667 | 18.3 | 0.8702 | 17.1 | 0.8726 | 17.6 | 0.8713 | 18.0 | 0.8761 | 18.1 |
| 0.8198 | 28.1 | 0.8453 | 23.2 | 0.8465 | 23.9 | 0.8444 | 25.4 | 0.8340 | 28.1 |
| 0.7901 | 33.5 | 0.8139 | 31.0 | 0.8089 | 31.6 | 0.8095 | 32.2 | 0.7955 | 35.5 |
| 0.7494 | 40.3 | 0.7866 | 34.5 | 0.7730 | 38.1 | 0.7884 | 36.2 | 0.7273 | 46.5 |
| 0.7112 | 46.0 | 0.7400 | 42.1 | 0.7252 | 45.2 | 0.7521 | 42.2 | 0.6423 | 58.4 |
| 0.6470 | 54.1 | 0.7104 | 46.1 | 0.6481 | 55.9 | 0.6352 | 57.5 | 0.5815 | 64.7 |
| 0.4948 | 69.2 | 0.6474 | 54.1 | 0.5688 | 63.7 | 0.5131 | 70.0 | 0.5090 | 71.5 |
|  |  | 0.5799 | 61.1 | 0.4976 | 71.0 |  |  |  |  |
|  |  | 0.4839 | 69.5 |  |  |  |  |  |  |
| $x_{2}{ }^{\circ}=0.6382$ |  | $x_{2}{ }^{0}=0.7158$ |  | $x_{2}{ }^{0}=0.8370$ |  | $x_{2}{ }^{0}=0.8877$ |  | $x_{2}{ }^{0}=0.9365$ |  |
| 0.8929 | 13.5 | 0.8910 | 15.0 | 0.9026 | 13.1 | 0.8977 | 15.4 | 0.8965 | 16.4 |
| 0.8702 | 20.4 | 0.8691 | 21.0 | 0.8654 | 25.1 | 0.8816 | 20.5 | 0.8757 | 22.4 |
| 0.8439 | 26.9 | 0.8498 | 25.8 | 0.8474 | 29.5 | 0.8535 | 28.6 | 0.8572 | 28.1 |
| 0.8086 | 33.8 | 0.8315 | 30.0 | 0.8307 | 33.7 | 0.8343 | 32.5 | 0.8390 | 32.1 |
| 0.7776 | 40.1 | 0.8039 | 35.7 | 0.7945 | 40.7 | 0.8057 | 38.5 | 0.8028 | 40.8 |
| 0.7463 | 44.0 | 0.7749 | 40.7 | 0.7678 | 45.5 | 0.7537 | 48.3 | 0.7690 | 45.2 |
| 0.7087 | 50.0 | 0.7255 | 48.7 | 0.7421 | 49.2 | 0.7005 | 56.5 | 0.7443 | 50.0 |
| 0.6571 | 57.9 | 0.6730 | 55.8 | 0.7228 | 52.5 | 0.6296 | 65.6 | 0.7162 | 54.6 |
| 0.6041 | 63.0 | 0.6073 | 64.2 | 0.6798 | 58.4 | 0.5444 | 76.4 | 0.6733 | 59.4 |
| 0.5426 | 68.7 | 0.5495 | 70.0 | 0.6347 | 64.0 | 0.4797 | 80.0 | 0.5857 | 70.8 |
| 0.4828 | 74.5 |  |  | 0.5371 | 75.2 |  |  | 0.5384 | 75.5 |

${ }^{a} x_{2}{ }^{\circ}=$ mole fraction of the fluorene in the benzene-free binary solid mixtures.
of the equation parameters and corresponding root mean square deviations for both of the liquidus curves of the three measured binary eutectic systems with benzene are presented in Table VI.
It can be noted that a good description has been obtained for every binary system. All deviations are in the range of $\sigma_{T}$ from 0.13 to $1.67^{\circ} \mathrm{C}$ for the Wilson equation and from 0.09 to $1.78^{\circ} \mathrm{C}$ for the UNIQUAC equation. The binary eutectic is the single condition at which a liquid is in equilibrium with two solid phases. Its temperature is a minimum on the liquidus curves of the $T-x_{i}$ diagram. The invariant eutectic point composition and temperature were $x_{1}=0.063, t=2.1$ ${ }^{\circ} \mathrm{C} ; x_{1}=0.112, t=-0.8^{\circ} \mathrm{C}$; and $x_{1}=0.060, t=2.8^{\circ} \mathrm{C}$ for the fluorene-benzene, dibenzofuran-benzene, and dibenzo-thiophene-benzene systems, respectively, as shown in Figures 1-3. Figures 2 and 3 show the solid-liquid equilibrium data obtained by two different experimental methods-dynamic, used in this work, and static, used in previous work (4), for the same binary systems. It can be noted that only one experimental point for dibenzothiophene-benzene obtained by the static method is far from the liquidus curve (see Figure 3). Generally, the results of the solid solubility correlation, obtained by using the Wilson and the UNIQUAC equations, are comparable with each other and not very far from the ideal solubility, which is shown in Table VII for the diben-

Table V. Equilibrium Data for the Ternary Syatem Benzene (1)-Dibenzothiophene (2)-Dibenzofuran (3)"

| $x_{1}$ | $\boldsymbol{t} /{ }^{\circ} \mathrm{C}$ | $x_{1}$ | $t /{ }^{\circ} \mathrm{C}$ | $x_{1}$ | $t /{ }^{\circ} \mathrm{C}$ | $x_{1}$ | $t /{ }^{\circ} \mathrm{C}$ | $x_{1}$ | $t /{ }^{\circ} \mathrm{C}$ |
| :---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{x}_{2}{ }^{\circ}=0.0401$ | $\boldsymbol{x}_{2}{ }^{\circ}=0.0453$ |  | $\boldsymbol{x}_{2}{ }^{\circ}=0.0830$ |  | $x_{2}{ }^{\circ}=0.0903$ |  | $x_{2}{ }^{\circ}=0.0 .1384$ |  |  |
| 0.8403 | 9.7 | 0.8070 | 15.1 | 0.8184 | 12.9 | 0.8229 | 11.8 | 0.7995 | 15.1 |
| 0.7827 | 20.5 | 0.7379 | 26.1 | 0.7260 | 27.6 | 0.8075 | 14.4 | 0.7158 | 27.6 |
| 0.7305 | 27.0 | 0.6372 | 39.1 | 0.6495 | 36.7 | 0.7527 | 22.9 | 0.6125 | 39.8 |
| 0.6790 | 34.0 | 0.4593 | 56.7 | 0.5501 | 46.8 | 0.7019 | 29.6 | 0.5761 | 43.6 |
| 0.5882 | 44.1 | 0.2429 | 70.5 | 0.4463 | 55.4 | 0.6551 | 35.6 | 0.5032 | 50.2 |
| 0.5148 | 51.5 | 0.1925 | 73.4 | 0.3604 | 62.8 | 0.6128 | 39.9 | 0.4240 | 56.2 |
| 0.4100 | 62.1 |  |  | 0.2683 | 70.5 | 0.5471 | 46.0 | 0.3424 | 62.4 |
|  |  |  |  |  |  | 0.4700 | 53.6 | 0.2781 | 70.0 |
|  |  |  |  |  |  | 0.3902 | 60.0 |  |  |


| $x_{2}{ }^{\circ}=0.1666$ |  | $x_{2}{ }^{\circ}=0.2080$ |  | $x_{2}{ }^{\circ}=0.2822$ |  | $x_{2}{ }^{0}=0.3532$ |  | $x_{2}{ }^{0}=0.3966$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.8160 | 11.7 | 0.8025 | 14.5 | 0.8097 | 14.9 | 0.8262 | 10.5 | 0.8192 | 12.2 |
| 0.7801 | 18.1 | 0.7586 | 21.5 | 0.7706 | 19.4 | 0.7318 | 25.5 | 0.7374 | 25.2 |
| 0.7374 | 24.9 | 0.7136 | 27.6 | 0.7118 | 27.6 | 0.6800 | 31.9 | 0.6883 | 31.7 |
| 0.6711 | 33.2 | 0.6695 | 33.2 | 0.6738 | 32.6 | 0.6049 | 40.1 | 0.6424 | 36.8 |
| 0.5770 | 43.3 | 0.6310 | 37.4 | 0.6383 | 36.7 | 0.5458 | 46.3 | 0.5970 | 41.9 |
| 0.4951 | 51.0 | 0.5612 | 45.0 | 0.5473 | 45.7 | 0.3990 | 58.6 | 0.5481 | 47.0 |
| 0.3975 | 59.3 | 0.4806 | 51.9 | 0.3610 | 60.8 | 0.2190 | 70.2 | 0.5170 | 49.2 |
| 0.2923 | 68.2 | 0.3946 | 58.9 | 0.3120 | 64.8 |  |  | 0.4158 | 57.5 |
|  |  | 0.3519 | 62.0 | 0.2409 | 70.5 |  |  | 0.3705 | 60.8 |
|  |  | 0.2551 | 70.1 |  |  |  |  | 0.2898 | 65.8 |
|  |  |  |  |  |  |  |  | 0.2231 | 70.2 |
| $x_{2}{ }^{0}=0.4402$ |  | $x_{2}{ }^{0}=0.5068$ |  | $x_{2}{ }^{\circ}=0.5485$ |  | $x_{2}{ }^{\circ}=0.5662$ |  | $x_{2}{ }^{\text {o }}=0.5990$ |  |
| 0.8161 | 13.3 | 0.8076 | 15.4 | 0.8201 | 14.2 | 0.8437 | 12.9 | 0.8364 | 13.5 |
| 0.7338 | 26.6 | 0.7648 | 22.7 | 0.8070 | 17.6 | 0.8163 | 17.7 | 0.7973 | 21.3 |
| 0.6672 | 35.7 | 0.7161 | 29.6 | 0.7601 | 24.1 | 0.7731 | 22.4 | 0.7222 | 32.4 |
| 0.5742 | 45.0 | 0.6563 | 36.9 | 0.7110 | 30.9 | 0.7212 | 30.0 | 0.6570 | 40.8 |
| 0.3848 | 60.2 | 0.5510 | 48.0 | 0.5838 | 45.5 | 0.6769 | 35.5 | 0.5851 | 49.0 |
| 0.2855 | 67.0 | 0.4584 | 56.4 | 0.4874 | 54.2 | 0.6121 | 42.8 | 0.5260 | 54.6 |
|  |  | 0.3426 | 65.1 | 0.3858 | 62.6 | 0.5633 | 47.5 | 0.4652 | 59.6 |
|  |  | 0.2553 | 70.5 | 0.2577 | 70.6 | 0.5385 | 49.7 | 0.3692 | 66.0 |
|  |  |  |  |  |  | 0.4900 | 54.7 | 0.2871 | 71.1 |
|  |  |  |  |  |  | 0.4519 | 57.9 |  |  |
|  |  |  |  |  |  | 0.3931 | 62.0 |  |  |
|  |  |  |  |  |  | 0.3452 | 65.1 |  |  |


| $x_{2}{ }^{\circ}=0.6572$ |  | $x_{2}{ }^{\circ}=0.7042$ |  | $x_{2}{ }^{\circ}=0.7623$ |  | $x_{2}{ }^{\circ}=0.8400$ |  | $x_{2}{ }^{\circ}=0.8630$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.8531 | 12.8 | 0.8574 | 13.8 | 0.8775 | 16.0 | 0.8928 | 15.5 | 0.8980 | 13.8 |
| 0.8173 | 19.8 | 0.8453 | 17.5 | 0.8378 | 24.4 | 0.8600 | 23.4 | 0.8601 | 22.7 |
| 0.7703 | 28.6 | 0.7961 | 28.2 | 0.7967 | 31.5 | 0.8156 | 32.1 | 0.8261 | 29.2 |
| 0.7104 | 37.5 | 0.7432 | 36.2 | 0.7368 | 40.4 | 0.7430 | 43.7 | 0.7881 | 35.3 |
| 0.6593 | 43.7 | 0.6753 | 45.2 | 0.6248 | 54.0 | 0.6796 | 51.4 | 0.7328 | 43.1 |
| 0.5732 | 56.6 | 0.6027 | 53.6 | 0.5041 | 65.8 | 0.5779 | 62.9 | 0.6610 | 51.2 |
| 0.4903 | 61.0 | 0.5251 | 60.6 | 0.3353 | 78.6 | 0.4489 | 73.2 | 0.5919 | 57.7 |
| 0.4122 | 66.6 | 0.4552 | 66.2 |  |  | 0.3463 | 79.5 | 0.4348 | 71.5 |
| 0.3230 | 72.1 | 0.3851 | 70.9 |  |  |  |  | 0.3252 | 78.9 |
| $x_{2}{ }^{\circ}=0.9132$ |  | $x_{2}{ }^{0}=0.9537$ |  |  |  |  |  |  |  |
| 0.8835 | 18.9 | 0.8986 | 15.8 |  |  |  |  |  |  |
| 0.8538 | 25.8 | 0.8955 | 16.8 |  |  |  |  |  |  |
| 0.8293 | 31.0 | 0.8315 | 31.5 |  |  |  |  |  |  |
| 0.7836 | 39.1 | 0.7594 | 43.7 |  |  |  |  |  |  |
| 0.7245 | 47.6 | 0.7138 | 48.8 |  |  |  |  |  |  |
| 0.6284 | 59.8 | 0.6752 | 53.7 |  |  |  |  |  |  |
| 0.5469 | 66.5 | 0.6460 | 57.5 |  |  |  |  |  |  |
| 0.3709 | 80.3 | 0.5838 | 63.0 |  |  |  |  |  |  |
|  |  | 0.5596 | 65.9 |  |  |  |  |  |  |
|  |  | 0.5324 | 67.5 |  |  |  |  |  |  |

${ }^{a} x_{2}{ }^{\circ}=$ mole fraction of the dibenzothiophene in the benzene-free binary solid mixtures.
zofuran-benzene system as an example. Values presented in this table have to be related to experimental values shown earlier in Table II.

When solid solutions occur, the solubility equation (eq 1 above) must be modified to include activity coefficients in the solid phase and must be applied to both components. The resulting equations, which hold when solid-phase transitions are absent, are

$$
\begin{align*}
& x_{1}^{*} \gamma_{1}^{*}=x_{1} \gamma_{1} \exp \left[\frac{\Delta H_{\mathrm{m} 1}}{R}\left(\frac{1}{T}-\frac{1}{T_{\mathrm{t} 1}}\right)+\right. \\
&\left.\frac{\Delta C_{p \mathrm{~m} 1}}{R}\left(\ln \frac{T_{\mathrm{t} 1}}{T}-\frac{T_{\mathrm{t} 1}}{T}+1\right)\right] \tag{5}
\end{align*}
$$

Table VI. Values of the Parameters and of the Root Mean Square Deviations of Temperature Obtained by Wilson and UNIQUAC Equations in Binary Eutectic Systems

| system | parameter |  | $\sigma_{T} /{ }^{\circ} \mathrm{C}$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} \text { Wilson } \\ g_{12}-g_{11} \\ \left(g_{12}-g_{22} /\right. \\ \left(\mathrm{Jmol}^{-1}\right) \\ \hline \end{gathered}$ | $\begin{aligned} & \text { UNIQUAC } \\ & \Delta U_{12}\left(\Delta U_{21}\right) / \\ & \left(J^{\left(\mathrm{mol}^{-1}\right)}\right) \end{aligned}$ |  |  |
|  |  |  | Wilson | $\begin{aligned} & \text { UNI- } \\ & \text { QUAC } \end{aligned}$ |
| fluorene (1)benzene (2) | $\begin{gathered} -1264.27 \\ (2210.32) \end{gathered}$ | $\begin{aligned} & 1462.73 \\ & (-738.38) \end{aligned}$ | 0.32 | 0.37 |
| dibenzofuran (1)benzene (2) | $\begin{array}{r} -1373.34 \\ (2354.80) \end{array}$ | $\begin{gathered} 1640.56 \\ (-852.64) \end{gathered}$ | 0.40 | 0.41 |
| dibenzothiophene (1)benzene (2) | $\begin{gathered} -1175.65 \\ (2828.56) \end{gathered}$ | $\begin{gathered} 1918.68 \\ (-830.52) \end{gathered}$ | 1.67 | 1.78 |
| benzene (1)fluorene (2) | $\begin{gathered} 78770.36 \\ (3053.71) \end{gathered}$ | $\begin{array}{r} 5443.90 \\ (-2877.67) \end{array}$ | 0.35 | 0.25 |
| benzene (1)dibenzofuran (2) | $\begin{aligned} & -2327.28 \\ & (100837.6) \end{aligned}$ | $\begin{array}{r} 6112.95 \\ (-2910.86) \end{array}$ | 0.13 | 0.09 |
| benzene (1)dibenzothiophene (2) | $\begin{array}{r} 8432769.6 \\ (5077.38) \end{array}$ | $\begin{aligned} & 1525.97 \\ & (2336122.1) \end{aligned}$ | 0.34 | 0.41 |



Figure 1. Solid-liquid phase diagram for fluorene (1)benzene (2). The lines are predicted by the Wilson equation.

$$
\begin{align*}
& x_{2}^{*} \gamma_{2}^{*}=x_{2} \gamma_{2} \exp \left[\frac{\Delta H_{\mathrm{m} 2}}{R}\left(\frac{1}{T}-\frac{1}{T_{\mathrm{t} 2}}\right)+\right. \\
&\left.\frac{\Delta C_{p \mathrm{~m} 2}}{R}\left(\ln \frac{T_{\mathrm{t} 2}}{T}-\frac{T_{\mathrm{t} 2}}{T}+1\right)\right] \tag{6}
\end{align*}
$$

where the mole fraction and activity coefficient of the solid phase are shown with an asterisk. Summing up the mole fractions leads to the two equations

$$
\begin{align*}
& x_{1}+x_{2}=\left(\gamma_{1}^{*} / \gamma_{1}\right) x_{1}^{*} \exp \left[\frac{\Delta H_{\mathrm{m} 1}}{R}\left(\frac{1}{T_{\mathrm{t} 1}}-\frac{1}{T}\right)-\right. \\
& \left.\frac{\Delta C_{p \mathrm{~m} 1}}{R}\left(\ln \frac{T_{\mathrm{t} 1}}{T}-\frac{T_{\mathrm{t} 1}}{T}+1\right)\right]+\left(\gamma_{2}^{*} / \gamma_{2}\right) x_{2}^{*} \times \\
& \quad \exp \left[\frac{\Delta H_{\mathrm{m} 2}}{R}\left(\frac{1}{T_{\mathrm{t} 2}}-\frac{1}{T}\right)-\frac{\Delta C_{p \mathrm{~m} 2}}{R}\left(\ln \frac{T_{\mathrm{t} 2}}{T}-\frac{T_{\mathrm{t} 2}}{T}+1\right)\right] \tag{7}
\end{align*}
$$



Figure 2. Solid-liquid phase diagram for dibenzofuran (1)benzene (2). The lines are predicted by the UNIQUAC equation. The solid circles represent experimental points published earlier (4).

Table VII. Values of the Calculated Temperatures and Activity Coefficients Obtained by Wilson and UNIQUAC Methods and Ideal Solubilities for the Dibenzofuran (1)-Benzene (2) System

| Wilson |  | UNIQUAC |  | ideal solubility $t /{ }^{\circ} \mathrm{C}$ |
| :---: | :---: | :---: | :---: | :---: |
| $t^{\text {calcd } /{ }^{\circ} \mathrm{C}}$ | $\gamma_{1}{ }^{\text {calcd }}$ | $t^{\text {calca } /{ }^{\circ} \mathrm{C}}$ | $\gamma_{1}{ }^{\text {calcd }}$ |  |
| 5.5 |  | 5.5 |  | $5.5^{\text {a }}$ |
| 2.8 | $1.023^{\text {a }}$ | 2.9 | $1.025^{\text {a }}$ | $1.3{ }^{\text {a }}$ |
| 0.9 | $1.019^{\text {a }}$ | 1.0 | $1.019^{\text {a }}$ | -0.2 ${ }^{\text {a }}$ |
| 0.2 | $1.016^{\text {a }}$ | 0.1 | $1.015^{\text {a }}$ | $-0.8{ }^{\text {a }}$ |
| -1.4 | 1.293 | -1.5 | 1.296 | -11.3 |
| 1.8 | 1.285 | 1.7 | 1.288 | -8.1 |
| 4.8 | 1.276 | 4.8 | 1.279 | -4.9 |
| 7.2 | 1.269 | 7.1 | 1.271 | -2.5 |
| 10.1 | 1.259 | 10.1 | 1.262 | 0.2 |
| 17.3 | 1.234 | 17.3 | 1.236 | 8.2 |
| 23.1 | 1.211 | 23.2 | 1.213 | 14.6 |
| 25.1 | 1.203 | 25.2 | 1.204 | 16.8 |
| 29.5 | 1.184 | 29.6 | 1.185 | 21.7 |
| 31.8 | 1.174 | 31.8 | 1.174 | 24.3 |
| 38.0 | 1.145 | 38.0 | 1.145 | 31.5 |
| 46.6 | 1.105 | 46.5 | 1.103 | 41.6 |
| 56.6 | 1.060 | 56.4 | 1.057 | 53.6 |
| 64.9 | 1.029 | 64.7 | 1.027 | 63.3 |
| 68.1 | 1.019 | 68.0 | 1.018 | 67.2 |
| 72.7 | 1.009 | 72.6 | 1.008 | 72.2 |
| 82.1 | 1.000 | 82.1 | 1.000 | 82.1 |

${ }^{a}$ Activity coefficients and ideal solubilities for benzene as a solute.

$$
\begin{gather*}
x_{1}^{*}+x_{2}^{* *}=\left(\gamma_{1} / \gamma_{1}^{* *}\right) x_{1} \exp \left[\frac{\Delta H_{\mathrm{m} 1}}{R}\left(\frac{1}{T}-\frac{1}{T_{\mathrm{t} 1}}\right)+\right. \\
\left.\frac{\Delta C_{p \mathrm{~m} 1}}{R}\left(\ln \frac{T_{\mathrm{t} 1}}{T}-\frac{T_{\mathrm{t} 1}}{T}+1\right)\right]+\left(\gamma_{2} / \gamma_{2}^{*}\right) x_{2} \times \\
\exp \left[\frac{\Delta H_{\mathrm{m} 2}}{R}\left(\frac{1}{T}-\frac{1}{T_{\mathrm{t} 2}}\right)+\frac{\Delta C_{p \mathrm{~m} 2}}{R}\left(\ln \frac{T_{\mathrm{t} 2}}{T}-\frac{T_{\mathrm{t} 2}}{T}+1\right)\right] \tag{8}
\end{gather*}
$$



Figure 3. Solid-liquid phase diagram for dibenzothiophene (1)-benzene (2). The lines are predicted by the Wilson equation. The solid circles represent experimental points published earlier (4).

Remembering that

$$
\begin{gather*}
x_{1}+x_{2}=1  \tag{9}\\
x_{1}^{*}+x_{2}^{*}=1 \tag{10}
\end{gather*}
$$

it is possible to minimize the following objective function:

$$
\begin{align*}
& \Omega=\sum\left[x_{i}^{\text {exppl }}-x_{i}^{\text {calcd }}\left(T, A_{1}, A_{2}, A_{3}, A_{4}\right)\right]^{2}+ \\
& \quad\left[x_{i}^{* * x p t l}-x_{i}^{* *} \text { calcd }\left(T, A_{1}, A_{2}, A_{3}, A_{4}\right)\right]^{2} \tag{11}
\end{align*}
$$

For an ideal solid phase ( $\gamma_{1}{ }^{*}=\gamma_{2}{ }^{*}=1$ ) the problem reduces to the minimization of $\Omega$ with two parameters ( $A_{1}, A_{2}$ ), using composition data from the liquidus curve. Using the experimental data of Sediawan et al. (1), with liquid-phase activity coefficients derived from the UNIQUAC and Wilson equations, the interaction parameters were fitted by this optimization technique. The effect of the $\alpha / \beta$ phase transition of dibenzothiophene had to be added for the diben-zothiophene-dibenzofuran system. The resulting parameters, calculated from the liquidus curves, are presented in Table VIII.

Table IX shows the experimental data of Sediawan (1) in the form of $x_{1}$ and $x_{1}{ }^{*}$ as functions of temperature for the solid solution region. The table also shows calculated temperatures: $T^{\text {cald }}$ (nonideal solid solution, four UNIQUAC parameters) and $T^{\text {ceald, }, \text { d }}$ (ideal solid solution, two UNIQUAC parameters). Values of the four parameters and standard deviations of temperature, obtained by th UNIQUAC model in every binary solid-solid system are shown in Table X.

Taking into consideration that the results of the solid solubility correlation obtained by using the Wilson and UNIQUAC equations were comparable with each other, calculations of ternary system phase diagrams have been continued with the UNIQUAC equation only. Using the parameters derived from the binary data, it is possible to

Table VIII. Values of the Parameters and of the Root Mean Square Deviations of Temperature Obtained by Wilson and UNIQUAC Equations in Binary Systems (Liquidus Curves) ${ }^{2}$

| syatem | parameter |  | $\sigma_{T} /{ }^{\circ} \mathrm{C}$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} \text { Wilson } \\ g_{12}-g_{11} \\ \left(g_{12}-g_{22} /\right. \\ \left(J \mathrm{~mol}^{-1}\right) \\ \hline \end{gathered}$ | $\begin{aligned} & \text { UNIQUAC } \\ & \Delta U_{12}\left(\Delta U_{21}\right) / \\ & \left(\mathrm{J}^{\mathrm{mol}}{ }^{-1}\right) \end{aligned}$ |  |  |
|  |  |  | Wilson | UNIQUAC |
| fluorene (1)dibenzofuran (2) | $\underset{(3420.17)}{2617.12}$ | $\begin{gathered} 1081.57 \\ (153.70) \end{gathered}$ | 1.9 | 2.2 |
| fluorene (1)dibenzothiophene (2) | $\begin{aligned} & 8009.28 \\ & (4381.22) \end{aligned}$ | $\begin{gathered} 129.57 \\ (1796.20) \end{gathered}$ | 1.5 | 3.0 |
| dibenzothiophene (1)fluorene (2) | $\begin{array}{r} 28721337.2 \\ (8346.55) \end{array}$ | $\begin{aligned} & 3046.89 \\ & \quad(51075.6) \end{aligned}$ | 0.3 | 0.2 |
| dibenzothiophene (1)dibenzofuran (2) | $\begin{gathered} 65651.02 \\ (2453.41) \end{gathered}$ | $\begin{aligned} & -185.45 \\ & (1827.92) \end{aligned}$ | 0.7 | 0.4 |
| dibenzofuran (1)dibenzothiophene (2) | $\begin{array}{r} 1795145.9 \\ (7693.86) \end{array}$ | $\begin{aligned} & 5683.57 \\ & (-2227.56) \end{aligned}$ | 2.8 | 0.1 |

Table IX. Equilibrium Data for the Binary System Dibenzothiophene (1)-Dibenzofuran (2) and Temperatures Calculated by the UNIQUAC Equation with Four and Two Parameters (Ideal Solid Phase)

| $x_{1}$ | $x_{1}{ }^{*}$ | $t /{ }^{\circ} \mathrm{C}$ | $t^{\text {calcd } /{ }^{\circ} \mathrm{C}}$ | $t^{\text {caled, id } * /{ }^{\circ} \mathrm{C}}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0.000 | 0.000 | 82.1 | 82.1 | 82.1 |
| 0.395 | 0.475 | 81.3 | 81.4 | 80.3 |
| 0.405 | 0.499 | 81.4 | 81.5 | 80.4 |
| 0.445 | 0.535 | 81.8 | 81.8 | 80.9 |
| 0.499 | 0.591 | 82.4 | 82.3 | 81.7 |
| 0.531 | 0.625 | 82.8 | 82.7 | 82.3 |
| 0.565 | 0.671 | 83.3 | 83.2 | 83.1 |
| 0.600 | 0.694 | 83.8 | 83.8 | 84.0 |
| 0.650 | 0.751 | 84.9 | 84.8 | 85.4 |
| 0.721 | 0.824 | 86.8 | 86.7 | 87.7 |
| 0.751 | 0.850 | 87.5 | 87.7 | 88.8 |
| 0.805 | 0.882 | 88.8 | 89.7 | 91.0 |
| 0.865 | 0.928 | 92.7 | 92.4 | 93.4 |
| 0.880 | 0.930 | 93.2 | 93.1 | 94.0 |
| 0.890 | 0.940 | 93.8 | 93.6 | 94.4 |
| 0.915 | 0.951 | 95.1 | 94.8 | 95.4 |
| 0.929 | 0.960 | 95.7 | 95.5 | 96.0 |
| 0.951 | 0.972 | 96.8 | 96.5 | 96.8 |
| 0.962 | 0.981 | 97.3 | 97.0 | 97.3 |
| 1.000 | 1.000 | 98.7 | 98.7 | 98.7 |

Table X. Values of the Parameters and of the Root Mean Square Deviations of Temperature Obtained by the UNIQUAC Equation in Binary Systems (Liquid and Solidus Curves) ${ }^{2}$

| system | parameter |  | $\sigma_{T} /{ }^{\circ} \mathrm{C}$ |
| :---: | :---: | :---: | :---: |
|  | $\overline{\Delta U_{12}\left(\Delta U_{21}\right) /}$ | $\begin{gathered} \Delta U_{12}^{b}\left(\Delta U_{21}^{b}\right) / \\ \left(\mathrm{J}^{\mathrm{mol}}{ }^{-1}\right) \end{gathered}$ |  |
| fluorene (1)dibenzofuran (2) | $\begin{aligned} & 2788.68 \\ & (-1921.60) \end{aligned}$ | $\begin{gathered} -131.37 \\ (453.08) \end{gathered}$ | 0.4 |
| fluorene (1)dibenzothiophene (2) | $\begin{aligned} & -1630.57 \\ & (3388.50) \end{aligned}$ | $\begin{gathered} -1516.13 \\ (3567.87) \end{gathered}$ | 0.1 |
| dibenzothiophene (1)dibenzofuran (2) | $\begin{array}{r} 1208.92 \\ (-1827.76) \end{array}$ | $\begin{aligned} & 3450.25 \\ & (-2509.60) \end{aligned}$ | 0.3 |
| ${ }^{a}$ Standard deviation phase. | $=\left[\sum _ { i = 1 } ^ { n } \left[\left(T_{i}^{c}\right.\right.\right.$ | $\left.\left.-T_{i}\right)^{2} / n\right]$ | Solid |

predict the solidus and liquidus curves in ternary, two-phase mixtures (solid solution-liquid). Taking into account 12 parameters ( 4 for every binary system), the equilibrium temperature can be predicted by solution of three equations similar to eqs 5 and 6 with $\sum x_{i}=1$ and $\sum x_{1}{ }^{*}=1$. In this way it is possible to calculate the temperature and composition of the solid phase for the given composition of the liquid phase. In every ternary system tested in this paper, we have assumed the four solid-phase parameters equal to zero for the two binary eutectic systems. The results of calculation at $50^{\circ} \mathrm{C}$ with values of binary eutectic parameters (aromatic compound-benzene liquidus curve) given in Table VI, parameters for solid-solid systems given in Table X, and four

Table XI. Values of the Temperatures Predicted by the UNIQUAC Equation for the System Benzene (1)-Fluorene (2)-Dibenzofuran (3) at $50{ }^{\circ} \mathrm{C}$

| $x_{2}$ | $x_{3}$ | $t^{\text {calcd } /{ }^{\circ} \mathrm{C}}$ | $x_{2}$ | $x_{3}$ | $t^{\text {calce } /}{ }^{\circ} \mathrm{C}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0432 | 0.4558 | 51.3 | 0.1860 | 0.2239 | 48.7 |
| 0.0727 | 0.4222 | 51.4 | 0.1967 | 0.1964 | 48.3 |
| 0.0982 | 0.3817 | 50.8 | 0.2127 | 0.1694 | 48.5 |
| 0.1133 | 0.3518 | 50.1 | 0.2174 | 0.1446 | 48.0 |
| 0.1154 | 0.3426 | 49.7 | 0.2351 | 0.1148 | 48.8 |
| 0.1381 | 0.3319 | 50.9 | 0.2382 | 0.0898 | 48.5 |
| 0.1529 | 0.2971 | 50.0 | 0.2356 | 0.0744 | 48.0 |
| 0.1642 | 0.2754 | 49.7 | 0.2426 | 0.0574 | 48.5 |
| 0.1749 | 0.2646 | 49.9 | 0.3253 | 0.0307 | 56.2 |

Table XII. Values of the Temperatures Predicted by the UNIQUAC Equation for the System Benzene (1)-Fluorene (2)-Dibenzothiophene (3) at $50{ }^{\circ} \mathrm{C}$

| $x_{2}$ | $x_{3}$ | $t^{\text {calcd } /{ }^{\circ} \mathrm{C}}$ | $x_{2}$ | $x_{3}$ | $t^{\text {calcd } /{ }^{\circ} \mathrm{C}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0080 | 0.2671 | 50.6 | 0.1186 | 0.2004 | 51.4 |
| 0.0237 | 0.2623 | 50.2 | 0.1347 | 0.1853 | 51.6 |
| 0.0391 | 0.2809 | 52.3 | 0.1511 | 0.1549 | 51.0 |
| 0.0442 | 0.2738 | 51.9 | 0.1631 | 0.1349 | 50.7 |
| 0.0572 | 0.2709 | 52.4 | 0.1784 | 0.1166 | 50.9 |
| 0.0663 | 0.2637 | 52.3 | 0.1851 | 0.1049 | 50.7 |
| 0.0671 | 0.2549 | 51.7 | 0.2033 | 0.0807 | 50.7 |
| 0.0936 | 0.2363 | 51.9 | 0.2193 | 0.0427 | 49.6 |
| 0.1025 | 0.2300 | 52.2 | 0.2291 | 0.0290 | 49.5 |
| 0.1096 | 0.2124 | 51.5 |  |  |  |

Table XIII. Values of the Temperatures Predicted by the UNIQUAC Equation for the System Benzene
(1)-Dibenzothiophene (2)-Dibenzofuran (3) at $50^{\circ} \mathrm{C}$

| $x_{2}$ | $x_{3}$ | $t^{\text {calld } /} /{ }^{\circ} \mathrm{C}$ | $x_{2}$ | $x_{3}$ | $t^{\text {calcd } /{ }^{\circ} \mathrm{C}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0188 | 0.4500 | 48.3 | 0.2113 | 0.2687 | 52.2 |
| 0.0211 | 0.4439 | 48.0 | 0.2357 | 0.2294 | 52.6 |
| 0.0407 | 0.4493 | 49.3 | 0.2450 | 0.2091 | 53.2 |
| 0.0444 | 0.4477 | 49.3 | 0.2604 | 0.1995 | 53.4 |
| 0.0681 | 0.4240 | 49.4 | 0.2576 | 0.0803 | 50.4 |
| 0.0825 | 0.4125 | 49.7 | 0.2562 | 0.0488 | 49.7 |
| 0.1036 | 0.3944 | 50.2 | 0.2831 | 0.0449 | 51.7 |
| 0.1406 | 0.3575 | 50.9 | 0.2657 | 0.0252 | 50.0 |
| 0.1752 | 0.3208 | 51.7 | 0.2766 | 0.0134 | 50.7 |
| 0.1935 | 0.2945 | 51.9 |  |  |  |

Table XIV. Values of the Standard Deviations of Temperature Obtained by the UNIQUAC Equation for Prediction of Isotherms in Ternary Systems

| system | $t /{ }^{\circ} \mathrm{C}$ | $\sigma_{T} /{ }^{\circ} \mathrm{C}$ |
| :---: | :---: | ---: |
| benzene (1)-fluorene (2)- | 15 | 17.3 |
| dibenzofuran (3) | 25 | 11.3 |
|  | 40 | 3.7 |
| benzene (1)-fluorene (2)- | 50 | 1.9 |
| dibenzothiophene (3) | 60 | 4.2 |
|  | 15 | 16.2 |
|  | 25 | 13.0 |
|  | 40 | 5.5 |
| benzene (1)-dibenzothiophene (2)- | 50 | 1.5 |
| dibenzofuran (3) | 60 | 3.8 |
|  | 70 | 5.0 |
|  | 15 | 18.4 |
|  | 25 | 13.3 |
|  | 40 | 5.5 |
|  | 50 | 1.7 |
|  | 60 | 3.3 |
|  | 70 | 6.7 |

parameters equal to zero are shown in Tables XI-XIII. Columns 1 and 2 of Tables XI-XIII list the experimental values of solution composition at $50^{\circ} \mathrm{C}$, and column 3 presents the predicted temperatures. The standard deviation of temperature was from 1.5 to $1.9^{\circ} \mathrm{C}$ in these ternary systems at $50^{\circ} \mathrm{C}$. Overly large values of the standard deviation were observed at higher and lower temperatures, especially at 15 ${ }^{\circ} \mathrm{C}\left(\bar{\sigma}=17.3^{\circ} \mathrm{C}\right)$ as shown in Table XIV. Figures $4-6$ show the isothermal lines constructed from data on compositions


Figure 4. Equilibria for benzene (1)-fluorene (2)-dibenzofuran (3). The second and last of these form solid solutions at any composition with point A characteristic for a rapidly changed liquidus curve and two binary eutectic points $\mathrm{E}_{1}$ and $\mathrm{E}_{2}$ for 1-2 and 1-3 systems, respectively.


Figure 5. Benzene (1)-fluorene (2)-dibenzothiophene (3). Two different crystal structures $\alpha$ and $\beta$ and the solid immiscibility gap are present between points $S_{\alpha}$ and $S_{\beta}$ and a minimum on the liquidus curves, changing the $\alpha$ for the $\beta$ form at point $A$. The binary eutectics are at points $E_{1}$ and $\mathrm{E}_{2}$ for 1-2 and 1-3, respectively.
of mixtures having the same saturation temperature. These diagrams are the contour diagrams of the equilibrium surface of the ternary systems in the triangular prism with temperature as the vertical axis. The isotherms are drawn for 15 , $25,40,50,60$, and $70^{\circ} \mathrm{C}$. In all the major regions hightemperature isotherms are spread over a large area as compared to low-temperature isotherms, indicating that the slope of the surface at lower temperature is steeper than that at high temperature.

In the first phase diagram (Figure 4) the isotherms represent the varying compositions of the liquid solution in equilibrium with solid solution for the benzene-fluorene-dibenzofuran system at the given temperature. At point A a rapid change in slope of the liquidus curve occurs in the binary system fluorene-dibenzofuran. This may be the reason for existence of the valley on the ternary surface, which is observed as a minimum of the isotherm curve.

Figure 5 shows isotherms for the benzene-fluorenedibenzothiophene system. Since pure fluorene and diben-


Figure 6. Benzene (1)-dibenzothiophene (2)-dibenzofuran (3). Two different crystal structures $\alpha$ and $\beta$ and the solid immiscibility gap are present between points $\mathrm{S}_{\alpha}$ and $\mathrm{S}_{\beta}$. Point A is characteristic for changing the $\alpha$ form for the $\beta$ form, and point $B$ is a minimum of the $T-x$ diagram on the liquidus curve. The binary eutectics are at points $\mathrm{E}_{1}$ and $\mathrm{E}_{2}$ for 1-2 and 1-3, respectively.
zothiophene have different crystal structures ( 16,17 ), it was observed (1) that the fluorene-dibenzothiophene system had a solid immiscibility gap between points $S_{\alpha}$ and $S_{\beta}$ on the solidus curve, and point $A$ was observed as a minimum on the liquidus curves, changing $\alpha$ for $\beta$ crystal structures on the liquidus curve. We observed two minima, connected with the eutectic points of fluorene-benzene and dibenzothiophenebenzene systems and the maximum of the isotherm connected with point A-changing of crystal structure $\alpha$ to $\beta$. The same explanation can be applied to the third system, presented in Figure 6, where point A represents the change of solid crystal structure from $\alpha$ to $\beta$ on the liquidus curve and point $\mathbf{B}$ is a minimum of the $T-x$ diagram. The eutectic valleys were found on both sides of the isotherms.

In summary, results of calculation show that the prediction of ternary system isotherms using the UNIQUAC binary syatem parameters is possible at moderate temperatures (40$60^{\circ} \mathrm{C}$ ) with average standard deviation $\bar{\sigma}_{T}=3.4^{\circ} \mathrm{C}$. At higher temperatures, the liquidus surfaces of the ternary systems are difficult to describe and the prediction of SLE, especially in the region of the eutectic valleys, is not generally satisfactory. It should be expected that the application of more parameters for two eutectic liquidus lines or two parts of liquidus characteristic for the $\alpha / \beta$ phase transition of the fluorene-dibenzothiophene system, for example, will make a better description of isotherm curves of such polynuclear aromatic systems possible. Additionally the effect of the immiscibility gap in the solid phase should be added to eqs 5 and 6. At lower temperatures the standard deviation of prediction is unacceptable. Perhaps the ternary systems under study have a tendency to form binary eutectics in equilibrium with solid solution and liquid at lower temperature. Generally, we may expect better predictions for ternary systems which have three pairs of binary eutectics or binary solid solutions.

Glossary
$a_{1}$
$A_{1}, A_{2}, A_{3}, A_{4}$ $\Delta C_{p \mathrm{~m} 1}$
$F$
$g_{i j}$ $\Delta H_{\mathrm{m} 1}$
$\Delta H_{\text {tr } 1}$
n
R
$\mathbf{S}_{\alpha}, \mathbf{S}_{\beta}$
$T / t$
$T_{\mathrm{m}} / t_{\mathrm{m}}$
$T_{t} / t_{\mathrm{t}}$
$T_{\mathrm{tr} 1} / t_{\mathrm{tr} 1}$
$\Delta T$
$T$ taled
$\Delta u_{i j}$
$V$
$x_{1}$
$x_{1}{ }^{\text {called }}$
$\Delta x_{1}$
$x_{i}{ }^{\circ}$
$w$
Greek Letters
$\alpha \quad$ crystal structures
$\beta \quad$ crystal structures
$\gamma_{i} \quad$ activity coefficient of component $i$
$\sigma_{T} \quad$ root mean square deviation of temperature
$\sigma_{x} \quad$ mean absolute deviation of composition
$\Omega \quad$ objective function for the least-squares optimization

## Literature Cited

(1) Sediawan, W. B.; Gupta, S.; McLaughlin, E. J. Chem. Eng. Data 1989, 34, 223.
(2) Choi, P. B.; Williams, C. P.; Buehring, K. G.; McLaughlin, E. J. Chem. Eng. Data 1985, 30, 403.
(3) Coon, J. E.; Troth, M.; McLaughlin, E. J. Chem. Eng. Data 1987, 32, 233.
(4) Coon, J. E.; Sediawan, W. B.; Auwaerter, J. E.; McLaughlin, E. J. Solution Chem. 1988, 17, 519.
(5) Gupta, A.; Gupta, S.; Groves, F. R. Jr.; McLaughlin, E. Fluid Phase Equilib. 1991, 64, 201.
(6) Gupta, A.;Domanska, U.;Groves, F. R., Jr.; McLaughlin, E. J. Chem. Eng. Data, submitted for publication.
(7) Domanska, U. Fluid Phase Equilib. 1986, 26, 201.
(8) Dale, G. H. Encyclopedia of Chemical Processing and Design; Dekker: New York, 1981; Vol. 13, p 464.
(9) Gomes, A.; Serrano, L.; Farelo, F. J. Chem. Eng. Data 1988, 33, 194.
(10) Jadhav, V. K.; Chivate, R.; Tavare, N. W. J. Chem. Eng. Data 1991, 36, 249.
(11) Oliver, G. D.; Eaton, M.; Huffman, H. M. J. Am. Chem. Soc. 1984, 70, 1502.
(12) Choi, P. B.; McLaughlin, E. AIChE J. 1983, 29, 150.
(13) Wilson, G. M. J. Am. Chem. Soc. 1964, 86, 127.
(14) Abrams, D. S.; Prausnitz, J. M. AIChE J. 1975, 21, 116.
(15) Neau, E.; Peneloux, A. Fluid Phase Equilib. 1981, 6, 1.
(16) Schudt, E.; Weitz, G. Structure Data of Organic Crystals; Springer-Verlag: Berlin-Heidelberg, 1971; Vol. III/5a.
(17) Schudt, E.; Heitz, G.; Stress, J. Structure Data of Organic Crystals; Springer-Verlag: Berlin-Heidelberg, 1985; Vol. III/10a.

Received for review April 14, 1992. Accepted August 3, 1992. We gratefully acknowledge support for this work from the Louisiana Center for Energy Studies through Grant No. 135-15-0153. U.D. also thanks the Chemical Engineering Department for additional financial support.


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